Strongly coupled gauge theories play an important role in different areas of physics. Quantum Chromodynamics is the non-Abelian SU(2) gauge theory that describes the strong interactions between quarks and gluons in particle physics. Some strongly correlated electron systems in condensed matter physics are described by Abelian U(1) gauge theories, and the toric code, a quantum information storage device, is an Abelian Z(2) gauge theory. Many non-trivial aspects of gauge theories are accessible to accurate numerical simulations on classical computers. However, at high fermion density or in out-of-equilibrium situations such simulations suffer from notorious sign problems that prevent the importance sampling underlying Monte Carlo calculations. Quantum simulators are accurately controllable quantum devices that do not suffer from sign problems, because their hardware is intrinsically quantum mechanical. For example, trapped ions that follow a laser-driven stroboscopic discrete time evolution through a sequence of quantum gate operations, have been used as a digital quantum simulator for particle-anti-particle pair creation. Analog quantum simulators, on the other hand, follow the continuous time-evolution of a tunable model Hamiltonian. Using ultra-cold atoms in optical lattices, analog quantum simulators have been designed for Abelian and non-Abelian gauge theories. Their experimental realization is a challenge for the foreseeable future, which holds the promise to access the real-time dynamics of string breaking, the out-of-equilibrium decay of a false vacuum, or the evolution of a chiral condensate after a quench, from first principles. Quantum link models which realize gauge theories including QCD, not with classical fields but with discrete quantum degrees of freedom, are ideally suited for implementation in ultracold quantum matter.